Instructions: Complete each of the following exercises for practice.

1. Compute the curl and divergence of the vector field $\mathbf{F}$.

   (a) $\mathbf{F} = \langle xy^2z^2, x^2yz^2, x^2y^2z \rangle$

   (b) $\mathbf{F} = \langle 0, x^3yz^2, y^4z^3 \rangle$

   (c) $\mathbf{F} = \langle xye^z, 0, yze^x \rangle$

   (d) $\mathbf{F} = \langle \sin(yz), \sin(zx), \sin(xy) \rangle$

   (e) $\mathbf{F} = \left\langle \frac{\sqrt{x}}{1+z}, \frac{\sqrt{y}}{1+x}, \frac{\sqrt{z}}{1+y} \right\rangle$

   (f) $\mathbf{F} = \langle \ln(2y+3z), \ln(x+3z), \ln(x+2y) \rangle$

   (g) $\mathbf{F} = \langle e^x \sin(y), e^y \sin(z), e^z \sin(x) \rangle$

   (h) $\mathbf{F} = \langle \arctan(xy), \arctan(yz), \arctan(zx) \rangle$

2. Determine whether or not $\mathbf{F}$ is conservative. If so, compute a potential f.

   (a) $\mathbf{F} = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

   (b) $\mathbf{F} = \langle xyz^4, x^2z^4, 4x^2yz^3 \rangle$

   (c) $\mathbf{F} = \langle z\cos(y), xz\sin(y), x\cos(y) \rangle$

   (d) $\mathbf{F} = \langle 1, \sin(z), y\cos(z) \rangle$

   (e) $\mathbf{F} = \langle e^{yz}, xze^{yz}, xye^{yz} \rangle$

   (f) $\mathbf{F} = \langle e^x \sin(yz), ze^x \cos(yz), ye^x \cos(yz) \rangle$

3. Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ for which $\mathrm{curl}(\mathbf{G}) = \mathbf{F}$? Why?

   (a) $\mathbf{F} = \langle x\sin(y), \cos(y), z - xy \rangle$

   (b) $\mathbf{F} = \langle x, y, z \rangle$

4. Prove every vector field of form $\mathbf{F}(x,y,z) = \langle f(x), g(y), h(z) \rangle$ is *irrotational* (i.e. $\mathrm{curl}(\mathbf{F}) = \mathbf{0}$).

5. Prove every vector field of form $\mathbf{F}(x,y,z) = \langle f(y,z), g(x,z), h(x,y) \rangle$ is *incompressible* (i.e. $\mathrm{div}(\mathbf{F}) = 0$).

6. Prove each of the following identities for scalar field $\alpha(x,y,z)$ vector fields $\mathbf{F}(x,y,z)$ and $\mathbf{G}(x,y,z)$.

   (a) $\mathrm{div}(\mathbf{F}+\mathbf{G}) = \mathrm{div}(\mathbf{F}) + \mathrm{div}(\mathbf{G})$

   (b) $\mathrm{curl}(\mathbf{F}+\mathbf{G}) = \mathrm{curl}(\mathbf{F}) + \mathrm{curl}(\mathbf{G})$

   (c) $\mathrm{div}(\alpha\mathbf{F}) = \alpha\,\mathrm{div}(\mathbf{F}) + \mathbf{F} \cdot \nabla\alpha$

   (d) $\mathrm{curl}(\alpha\mathbf{F}) = \alpha\,\mathrm{curl}(\mathbf{F}) + (\nabla\alpha) \times \mathbf{F}$

   (e) $\mathrm{div}(\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \mathrm{curl}(\mathbf{F}) - \mathbf{F} \cdot \mathrm{curl}(\mathbf{G})$

   (f) $\mathrm{curl}(\mathrm{curl}(\mathbf{F})) = \nabla(\mathrm{div}\,\mathbf{F}) - \nabla^2\mathbf{F}$

7. Prove every continuous function $f(x,y,z)$ is the divergence of some vector field.